Correction - Maple - Chapitre 4

```
> restart :
```

Introduction

```
> n :=10 : a :=array(0..n) :
  P :=add(a[k]*x**k,k=0..n) :
> coeff(P,x,4);
```

$$a_4$$

```
> collect(P,x);
```

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10}$$

```
> degree(P,x);
```

$$10$$

Exercice 17

1.

```
> n :=10 :
  P :=expand((x+y)^n);
```

$$P := x^{10} + 10 x^9 y + 45 x^8 y^2 + 120 x^7 y^3 + 210 x^6 y^4 + 252 x^5 y^5 + 210 x^4 y^6 + 120 x^3 y^7 + 45 x^2 y^8 + 10 x y^9 + y^{10}$$

```
> coeff(P,x,3);
```

$$120 y^7$$

2.

```
> binomial(10,3);
```

$$120$$

3.

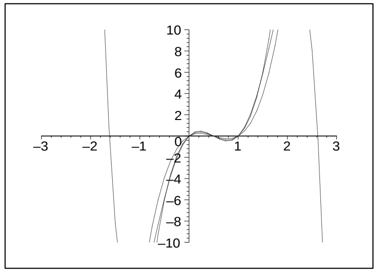
```
> restart :
  Bernstein :=proc(f,n)
  local k;
  add(binomial(n,k)*f(k/n)*x^k*(1-x)^(n-k),
    k=0..n);
  end proc :
```

4.

```
> f :=x->sin(2*Pi*x) :
> with(plots) :
```

Warning, the name changecoords has been redefined

```
> animation :=proc(f,Nmax)
  local G,n,g;
  G :=[seq(0,k=1..Nmax)] :
  for n from 1 to Nmax do;
  g :=t->subs(x=t,Bernstein(f,n));
  G[n] :=plot(g,-3..3,-10..10);
  od :
  display(G);
  end proc :
> animation(f,5);
```



Exercice 18

```
> P :=x^5+x+1 :
  Q :=x^5+x^13-1 :
> fsolve(P,x);
```

$$-0.7548776662$$

```
> fsolve(Q);
```

$$0.9203182932$$

Exercice 19

1.

```
> for n to 10 do
  expand(cos(n*arccos(x)));
  od;
```

$$x$$
$$2 x^2 - 1$$
$$4 x^3 - 3 x$$
$$8 x^4 - 8 x^2 + 1$$
$$16 x^5 - 20 x^3 + 5 x$$
$$32 x^6 - 48 x^4 + 18 x^2 - 1$$
$$64 x^7 - 112 x^5 + 56 x^3 - 7 x$$
$$128 x^8 - 256 x^6 + 160 x^4 - 32 x^2 + 1$$
$$256 x^9 - 576 x^7 + 432 x^5 - 120 x^3 + 9 x$$
$$512 x^{10} - 1280 x^8 + 1120 x^6 - 400 x^4 + 50 x^2 - 1$$

ou

$$t$$

```
> P :=array(1..10) :
  for n to 10 do
  P[n] :=subs(cos(x)=x,expand(cos(n*x)));
  od;
```

$$P_1 := x$$
$$P_2 := 2 x^2 - 1$$
$$P_3 := 4 x^3 - 3 x$$
$$P_4 := 8 x^4 - 8 x^2 + 1$$
$$P_5 := 16 x^5 - 20 x^3 + 5 x$$
$$P_6 := 32 x^6 - 48 x^4 + 18 x^2 - 1$$
$$P_7 := 64 x^7 - 112 x^5 + 56 x^3 - 7 x$$
$$P_8 := 128 x^8 - 256 x^6 + 160 x^4 - 32 x^2 + 1$$
$$P_9 := 256 x^9 - 576 x^7 + 432 x^5 - 120 x^3 + 9 x$$
$$P_{10} := 512 x^{10} - 1280 x^8 + 1120 x^6 - 400 x^4 + 50 x^2 - 1$$

2. $\cos(\theta n) = 0$ admet comme solution $n\theta \in \frac{\pi}{2} + \pi\mathbb{Z}$. Si on pose $\theta_k = \frac{\pi}{2n} + \frac{k\pi}{n}$ et $x_k = cos(\theta_k)$ pour $k = 0, \ldots, n-1$, on obtient n racines distinctes de P_n. Comme P_n est de degré n, on a toutes les racines de P_n.

3.

```
> rac :=[seq(0,k=1..13)] :
  for n to 13 do
  theta[n] :=[seq(Pi/(2*n)+k*Pi/n,k=0..n-1)];
  rac[n] :=map(cos,theta[n]) :
  od :
```

4.

```
> vérif :=[seq(0,k=1..n)] :
  for n from 1 to 10 do
  vérif[n] :=[seq(
    simplify(subs(x=rac[n][k],P[n])),
    k=1..nops(rac[n]) )] :
  od;
```

$$vrif_1 := [0]$$
$$vrif_2 := [0, 0]$$
$$vrif_3 := [0, 0, 0]$$
$$vrif_4 := [0, 0, 0, 0]$$
$$vrif_5 := [0, 0, 0, 0, 0]$$
$$vrif_6 := [0, 0, 0, 0, 0, 0]$$
$$vrif_7 := [0, 0, 0, 0, 0, 0, 0]$$
$$vrif_8 := [0, 0, 0, 0, 0, 0, 0, 0]$$
$$vrif_9 := [0, 0, 0, 0, 0, 0, 0, 0, 0]$$
$$vrif_{10} := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

Exercice 20

```
> symétrique :=proc(P)
  local k,n,c,Q;
  n :=degree(P);
  c :=[seq(coeff(P,x,k),k=0..n)];
  c[2];
  Q :=add(c[n-k+1]*x^k,k=0..n) :
  end proc :
```

```
> symétrique(1+2*x^2-x);
```

$$2 - x + x^2$$

```
> Q5 :=symétrique(P[5]);
```

$$Q5 := 16 - 20\,x^2 + 5\,x^4$$

```
> solve(Q5);
```

$$-\frac{\sqrt{50+10\sqrt{5}}}{5},\ \frac{\sqrt{50+10\sqrt{5}}}{5},$$
$$-\frac{\sqrt{50-10\sqrt{5}}}{5},\ \frac{\sqrt{50-10\sqrt{5}}}{5}$$

Exercice 21

```
> restart : P :=2*x^3-3*x^2-x-3 :
```

```
> Pprim :=diff(P,x);
```

$$Pprim := 6\,x^2 - 6\,x - 1$$

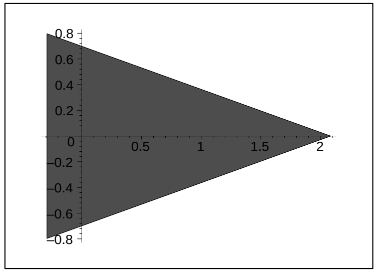
```
> S :=[solve(P)] :
  A :=[Re(S[1]),Im(S[1])] :evalf(A);
  B :=[Re(S[2]),Im(S[2])] :evalf(B);
  C :=[Re(S[3]),Im(S[3])] :evalf(C);
```

$$[2.084900317, 0.]$$
$$[-0.2924501587, 0.7961983185]$$
$$[-0.2924501587, -0.7961983185]$$

```
> Sprim :=[solve(Pprim)];
  E :=[Re(Sprim[1]),Im(Sprim[1])];
  F :=[Re(Sprim[2]),Im(Sprim[2])];
```

$$Sprim := [\frac{1}{2} + \frac{\sqrt{15}}{6},\ \frac{1}{2} - \frac{\sqrt{15}}{6}]$$
$$E := [\frac{1}{2} + \frac{\sqrt{15}}{6},\ 0]$$
$$F := [\frac{1}{2} - \frac{\sqrt{15}}{6},\ 0]$$

```
> with(plots) :
  triangle :=polygonplot([A,B,C],
    color=red) :
  pointsbase :=plot([E,F],color=blue,
    thickness=2) :
  display([triangle,pointsbase]);
Warning, the name changecoords has been redefined
```




```
> # milieu de [EF]
  centre :=(E+F)/2;
```

$$centre := [\frac{1}{2}, 0]$$

On cherche alors une ellipse d'équation

$$\frac{(x-1/2)^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Elle passe par les milieux des cotés.

```
> # milieu de [AB]
  Ic :=(A+B)/2 :

  # milieu de [BC]
  Ia :=(B+C)/2 :

  # milieu de [CA]
  Ib :=(A+C)/2 :
```

```
> eq :=(x-1/2)^2/a^2+y^2/b^2-1 :
  eq1 :=subs(x=Ic[1],y=Ic[2],eq) :
  eq2 :=subs(x=Ia[1],y=Ia[2],eq) :
  fsolve({eq1,eq2},{a,b});
```

$$\{a = -0.7924501587, b = -0.4596853134\}$$

```
> assign(%) : a :=abs(a) : b :=abs(b) :
```

On vérifie que l'ellipse passe par Ic

```
> evalf(subs(x=Ic[1],y=Ic[2],eq));
```

$$0.$$

On vérifie que l'ellipse est tangente aux cotés. On rappelle que l'équation de la tangente en un point (x_0,y_0) est :

$$y_0 y/b^2 + x_0 x/a^2 = 1.$$

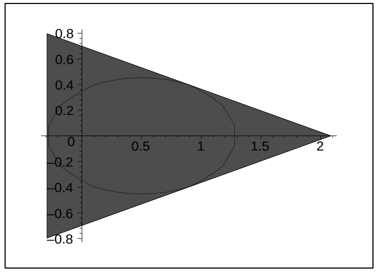
```
> vérif_tangente :=proc(A,B,Ix,a,b)
  local eq1, eq2, x0,y0, tgte,
   Delta, droite, pt;
  # équation de tangente en milieu
  x0 :=evalf(Ix[1]);
  y0 :=evalf(Ix[2]);
  tgte :=y0*y/b^2+(x0-1/2)*(x-1/2)/a^2-1;
  eq1 :=simplify(%/coeff(%,x,1));
  # équation de la droite AB
  pt :=evalf(A);
  Delta :=evalf(B-A);
  droite :=Delta[1]*(y-pt[2])-Delta[2]
   *(x-pt[1]);
  eq2 :=simplify(%/coeff(%,x,1));
  simplify(eq1-eq2);
  end proc :
```

```
> # En Ic
  vérif_tangente(B,C,Ia,a,b);
  # En Ia
  vérif_tangente(A,B,Ic,a,b);
  # En Ic
  vérif_tangente(A,C,Ib,a,b);
```

$$0$$
$$0.1000000000\,10^{-9}\,x$$
$$0.1000000000\,10^{-9}\,x$$

```
> Ellipse :=implicitplot(eq,x=-0.5..2,
   y=-2..2,color=blue) :
```

```
> display([triangle,pointsbase,Ellipse]);
```



Exercice 22

```
> restart :
```

```
> a :=0 :
  b :=1 :
  N :=30 :
  L :=array(0..N) :
  for k from 0 to N do L[k] :=a+k*(b-a)/N : od :
  f :=x->exp(-(x-0.5)^2) :
  fL :=map(f,L) :
  eval(L) :
```

On veut $diff\_div[k,j] = f[x_{k-j},...,x_k]$.

```
> diff_div :=array(0..N,0..N);
  for j from 0 to N do
  for k from j to N do
  if j=0 then diff_div[k,j] :=fL[k] :
  else
  diff_div[k,j] :=(diff_div[k,j-1]
   -diff_div[k-1,j-1])/(L[k]-L[k-j]) :
  fi :
  od :
  od :
```

$$diff\_div := \text{array}(0..30,\ 0..30,\ [\,])$$

```
> Lagrange :=proc(L,fL,N)
  local k,P,f,prod :
  P :=array(0..N) :
  P[0] :=fL[0] :
  prod :=1 :
  for k from 1 to N do
  prod :=prod*(x-L[k-1]) :
  P[k] :=P[k-1]+diff_div[k,k]*prod;
  od;
  sort(expand(P[N]));
  end proc :
```

```
> Lag :=Lagrange(L,fL,N);
```

$$\begin{aligned}Lag := &-0.2018874286\,10^{11}\,x^{30} + 0.3028311429\,10^{12}\,x^{29}\\ &-0.2167845063\,10^{13}\,x^{28} + 0.9858256882\,10^{13}\,x^{27}\\ &-0.3198045953\,10^{14}\,x^{26} + 0.7878936211\,10^{14}\,x^{25}\\ &-0.1532386903\,10^{15}\,x^{24} + 0.2414562270\,10^{15}\,x^{23}\\ &-0.3138812466\,10^{15}\,x^{22} + 0.3410568330\,10^{15}\,x^{21}\\ &-0.3127139312\,10^{15}\,x^{20} + 0.2436109085\,10^{15}\,x^{19}\\ &-0.1620095216\,10^{15}\,x^{18} + 0.9225681228\,10^{14}\,x^{17}\\ &-0.4505509692\,10^{14}\,x^{16} + 0.1887384374\,10^{14}\,x^{15}\\ &-0.6774280653\,10^{13}\,x^{14} + 0.2078221914\,10^{13}\,x^{13}\\ &-0.5428393055\,10^{12}\,x^{12} + 0.1200706568\,10^{12}\,x^{11}\\ &-0.2232621255\,10^{11}\,x^{10} + 0.3456513197\,10^{10}\,x^{9}\\ &-0.4400099550\,10^{9}\,x^{8} + 0.4530298675\,10^{8}\,x^{7}\\ &-0.3690084438\,10^{7}\,x^{6} + 230634.8236\,x^{5}\\ &-10580.80645\,x^{4} + 331.6647244\,x^{3} - 6.692742975\,x^{2}\\ &+0.8323440780\,x + 0.7788007831\end{aligned}$$

```
> Lag2 :=sort(interp(L,fL,x));
```

$$\begin{aligned}Lag2 := &-0.2027800440\,10^{11}\,x^{30} + 0.3041478914\,10^{12}\,x^{29}\\ &-0.2177113169\,10^{13}\,x^{28} + 0.9899689877\,10^{13}\,x^{27}\\ &-0.3211257330\,10^{14}\,x^{26} + 0.7910924680\,10^{14}\,x^{25}\\ &-0.1538500749\,10^{15}\,x^{24} + 0.2424028478\,10^{15}\,x^{23}\\ &-0.3150903951\,10^{15}\,x^{22} + 0.3423478103\,10^{15}\,x^{21}\\ &-0.3138770750\,10^{15}\,x^{20} + 0.2445013621\,10^{15}\,x^{19}\\ &-0.1625915376\,10^{15}\,x^{18} + 0.9258260662\,10^{14}\,x^{17}\\ &-0.4521153046\,10^{14}\,x^{16} + 0.1893829003\,10^{14}\,x^{15}\\ &-0.6797036049\,10^{13}\,x^{14} + 0.2085091824\,10^{13}\,x^{13}\\ &-0.5446059346\,10^{12}\,x^{12} + 0.1204555382\,10^{12}\,x^{11}\\ &-0.2239673808\,10^{11}\,x^{10} + 0.3467279381\,10^{10}\,x^{9}\\ &-0.4413622015\,10^{9}\,x^{8} + 0.4544045267\,10^{8}\,x^{7}\\ &-0.3701148489\,10^{7}\,x^{6} + 231318.7013\,x^{5}\\ &-10611.86294\,x^{4} + 332.6312265\,x^{3}\\ &-6.710929234\,x^{2} + 0.8324975268\,x + 0.7788007831\end{aligned}$$

$$115.3196829$$

```
> verif :=array(0..N) :
  for i from 0 to 3 do
  verif[i] :=subs(x=L[i],Lag)-fL[i]; od :
  seq(verif[i],i=0..3);
```

$$0.,\ 0.,\ -0.2\,10^{-9},\ 0.26\,10^{-8}$$

Exercice 23

```
> P :=1-x^2+3*x^4-5*x^7 :
> fsolve(P,x);
```

$$0.8780839519$$

```
> subs(x=%,P);
```

$$-0.1\,10^{-8}$$

Exercice 24

```
> F :=exp(-x^2) :
> N :=3 :
  f :=array(0..N) :
  f[0] :=F;
  for n from 1 to N do
  f[n] :=diff(F,x$n);
  od;
```

$$f_0 := e^{(-x^2)}$$
$$f_1 := -2\,x\,e^{(-x^2)}$$
$$f_2 := -2\,e^{(-x^2)} + 4\,x^2\,e^{(-x^2)}$$
$$f_3 := 12\,x\,e^{(-x^2)} - 8\,x^3\,e^{(-x^2)}$$

```
> N :=3 :
  h :=[seq(0,k=1..N)] :
  for n from 1 to N do
  h[n] :=simplify(exp(x^2)*f[n]) :
  od;
```

$$h_1 := -2\,x$$
$$h_2 := -2 + 4\,x^2$$
$$h_3 := -4\,x\,(-3 + 2\,x^2)$$

```
> solve(h[1],x);
```

$$0$$

```
> solve(h[2],x);
```

$$\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2}$$

```
> solve(h[3],x);
```

$$0,\ \frac{\sqrt{6}}{2},\ -\frac{\sqrt{6}}{2}$$

```
> int(h[3]-h[2],x=-1..1);
```

$$\frac{4}{3}$$

Exercice 25

```
> restart :
  P :=x^4+(3-4*a)*x^3+(-9*a+6*a^2+2)*x^2
  + (-4*a^3-4*a+9*a^2)*x + 2*a^2-3*a^3+a^4;
```

$$P := x^4 + (3 - 4\,a)\,x^3 + (-9\,a + 6\,a^2 + 2)\,x^2$$
$$+(-4\,a^3 - 4\,a + 9\,a^2)\,x + 2\,a^2 - 3\,a^3 + a^4$$

```
> factor(P);
```

$$(x - a + 2)\,(x - a + 1)\,(x - a)^2$$

```
> solve(P,x);
```

$$a - 1,\ a - 2,\ a,\ a$$

```
> expand(P);
```

$$x^4 + 3\,x^3 - 4\,x^3\,a - 9\,x^2\,a + 6\,x^2\,a^2 + 2\,x^2$$
$$-4\,x\,a^3 - 4\,x\,a + 9\,x\,a^2 + 2\,a^2 - 3\,a^3 + a^4$$

```
> collect(P,a);
```

$$a^4 + (-4\,x - 3)\,a^3 + (6\,x^2 + 9\,x + 2)\,a^2$$
$$+(-4\,x^3 - 9\,x^2 - 4\,x)\,a + x^4 + 3\,x^3 + 2\,x^2$$

```
> a :=-1 : subs(x=1+sqrt(3),P);
```

$$(1 + \sqrt{3})^4 + 7\,(1 + \sqrt{3})^3 + 17\,(1 + \sqrt{3})^2 + 23 + 17\,\sqrt{3}$$

```
> simplify(%);
```

$$189 + 109\,\sqrt{3}$$

```
> a :='a';
```

$$a := a$$

```
> int(P,x=-1..1);
```

$$\frac{26}{15} - 6\,a + 8\,a^2 - 6\,a^3 + 2\,a^4$$

Exercice 26

```
> N :=10 :
  L :=array(0..N) :
  L[0] := 1 :
  for i from 1 to N do
  L[i] :=simplify((1/i!)*exp(x)*
    diff(exp(-x)*x^i,x$i)) :
  od :
> produit_scalaire :=array(0..N,0..N) :
  for i from 0 to N do
  for j from 0 to N do
  if j<>i then
  produit_scalaire[i,j] :=int(exp(-x)*L[i]*L[j],
    x=0..infinity);
  fi;
  od;
  od;
  eval(produit_scalaire);
> solutions :=[seq(0,k=1..N)] :
  for i from 1 to N do
  solutions[i] :=sort([fsolve(L[i])]);
  od :

  racines :=incluses :
  for i from 1 to N-1 do
  if solutions[i][1] < solutions[i+1][1] then
  racines :=non_incluses :
  fi :
  for j from 1 to i do
  if solutions[i][j] > solutions[i+1][j+1] then
  racines :=non_incluses :
  fi :
  od :
  od :
  racines;
```

Exercice 27

```
> restart :
  P := a+b*x+c*x^2+d*x^3 :
> Itg :=array(0..3) :
> for i from 0 to 3 do
  Itg[i] :=int(x^i*P,x=0..1);
  od :
> solve({Itg[0]-1,Itg[1],Itg[2],Itg[3]},{a,b,c,d});
```

$$\{a = 16,\ b = -120,\ c = 240,\ d = -140\}$$

Pour obtenir la valeur de P, on peut faire

```
> subs(a=16, b=-120, c=240, d=-140,P);
```

$$16 - 120\,x + 240\,x^2 - 140\,x^3$$

ou

```
> assign(%%) :
> P ;
```

$$16 - 120\,x + 240\,x^2 - 140\,x^3$$

Exercice 28

```
> restart :
> eg :=(a^2-1)*t^2+b*(1-t*a*c-b*t^ 2)
  =a*(1-c*t^2)+1-2*c^2*t :

> nops(eg) ; op(1,eg) ; op(2,eg) ;
```

$$2$$

$$(a^2 - 1)\,t^2 + b\,(1 - t\,a\,c - b\,t^2)$$

$$a\,(1 - c\,t^2) + 1 - 2\,c^2\,t$$

```
> P :=op(1,eg)-op(2,eg) ;
```

$$P := (a^2 - 1)\,t^2 + b\,(1 - t\,a\,c - b\,t^2)$$
$$-a\,(1 - c\,t^2) - 1 + 2\,c^2\,t$$

```
> collect(P,t) ;
```

$$(a^2 - 1 + a\,c - b^2)\,t^2 + (2\,c^2 - b\,a\,c)\,t - a + b - 1$$

```
> c2 :=coeff(P,t,2) ;
  c1 :=coeff(P,t,1) ;
  c0 :=coeff(P,t,0) ;
```

$$c2 := a^2 - 1 + a\,c - b^2$$

$$c1 := 2\,c^2 - b\,a\,c$$

$$c0 := -a + b - 1$$

```
> solve({c2,c1,c0},{a,b,c}) ;
```

$$\{a = -1,\ c = 0,\ b = 0\},$$
$$\{a = 2,\ b = 3,\ c = 3\},$$
$$\{a = -2,\ b = -1,\ c = 1\}$$